Annex C

Modeling species presence-absence in the ecological niche theory framework using shape-constrained generalized additive models

L. Citores, L. Ibaibarriaga, D.J. Lee, M.J. Brewer, M. Santos, G. Chust

Imposing concavity constraints in the linear predictor scale, with a logit link function, result in unimodal probability curves.

Let $p(x), x \in \mathbb{R}$ be the estimated response probability curve obtained when imposing concavity restrictions in the linear predictor scale, with $h = logit$ as link function. Then,

$h(p(x))$ is concave $\overset{1)}{\Rightarrow}$ $h(p(x))$ is quasiconcave $\overset{2)}{\Rightarrow}$ $h^{-1}(h(p(x))) = p(x)$ is quasiconcave $\overset{3)}{\Rightarrow}$ $p(x)$ is unimodal

1)

Definition 2.2 (Avriel et al. 1988): A function f defined on the convex set $C \in \mathbb{R}^n$ is called concave if for every $x_1, x_2 \in C$ and $0 \leq \lambda \leq 1$ we have $f(\lambda(x_1 + (1-\lambda)x_2)) \geq \lambda f(x_1) + (1-\lambda) f(x_2)$.

Theorem 3.1 (Avriel et al. 1988): Let f be defined on the convex set $C \in \mathbb{R}^n$. It is a quasiconcave function if and only if $f(\lambda(x_1 + (1-\lambda)x_2)) \geq min(f(x_1), f(x_2))$ for every $x_1, x_2 \in C$ and $0 \leq \lambda \leq 1$.

It is clear that a concave function is also quasiconcave (not vice versa), since

$f(\lambda(x_1 + (1-\lambda)x_2)) \geq \lambda f(x_1) + (1-\lambda) f(x_2) \geq min(f(x_1), f(x_2))$

2)

Proposition 3.2 (Avriel et al. 1988): Let ϕ be a quasiconcave function defined on $C \in \mathbb{R}^n$ and let f be a nondecreasing function on $D \in \mathbb{R}$, containing the range of ϕ. Then the composite function $f\phi(x)$ is also quasiconcave.

Thus, this proof can be generalized to any link function h whose inverse h^{-1} is nondecreasing (in our case $h(p(x))$ is quasiconcave, and h^{-1} (antilogit function) is a non decreasing function obtaining that the composite $h^{-1}(h(p(x))) = p(x)$ is quasiconcave).

3)

Proposition 3.8 (Avriel et al. 1988): Let f be defined on the interval $C \in \mathbb{R}$ and suppose that it attains its maximum at a point $x^* \in C$. Then f is quasiconcave if and only if it is unimodal on C.

Proofs for Theorem 3.1, Proposition 3.2 and Proposition 3.8 can be found in Avriel et al. (1988).

References

Avriel, M., Diewert, W.E., Schaible, S., Zang, I. Generalized Concavity. *Plenum Press*, 1988.