Deep GEM-Based Network for Weakly Supervised 
UWB Ranging Error Mitigation

Yuxiao Li∗, Santiago Mazuelas†, and Yuan Shen∗

∗ Department of Electronic Engineering, Tsinghua University, Beijing, China
† BCAM-Basque Center for Applied Mathematics, and IKERBASQUE-Basque Foundation for Science, Bilbao, Spain
Email: li-yx18@mails.tsinghua.edu.cn, smazuelas@bcamath.org, shenyuan_e@tsinghua.edu.cn

Abstract—Ultra-wideband (UWB)-based techniques, while becoming mainstream approaches for high-accurate positioning, tend to be challenged by ranging bias in harsh environments. The emerging learning-based methods for error mitigation have shown great performance improvement via exploiting high semantic features from raw data. However, these methods rely heavily on fully labeled data, leading to a high cost for data acquisition. We present a learning framework based on weak supervision for UWB ranging error mitigation. Specifically, we propose a deep learning method based on the generalized expectation-maximization (GEM) algorithm for robust UWB ranging error mitigation under weak supervision. Such method integrate probabilistic modeling into the deep learning scheme, and adopt weakly supervised labels as prior information. Extensive experiments in various supervision scenarios illustrate the superiority of the proposed method.

Index Terms—UWB radio, ranging error mitigation, weakly supervised Learning, generalized expectation-maximization algorithm, deep learning

I. INTRODUCTION

Location-awareness has been playing an increasingly essential role in the new generation of wireless networks [1], [2], wherein centimeter-level precise positioning is required. Among the related approaches, Ultra-wideband (UWB)-based technique has continued to attract most of the research interest due to the wide bandwidth of more than 500 MHz in the 3.1 – 10.6 GHz band [3]. However, its performance is often degraded in harsh environments due to multipath effects [4] and non-line-of-sight (NLOS) conditions [5].

Extensive error mitigation techniques have been proposed based on both statistical models and learning techniques [6], [7]. Conventional model-driven methods mainly use a simplified propagation model [8] or manually extracted features [9], [10]. Such models and features are often inefficient to derive and can hardly reflect signal characteristics in complex environment [11], [12]. Deep learning (DL) methods, on the other hand, learn high semantic features from signals efficiently and in turn can achieve significant performance improvements [13]–[17]. These methods require large amount of labeled data, where both the received signals and their corresponding ranging error are known. As a result, the scarcity of labeled data has become a severe issue for developing efficient and robust solutions. The acquisition of fully and accurately labeled data, especially for many wireless localization scenarios, is often infeasible and at great cost of time, man-hour and money.

In contrast to the labeled data, unlabeled data are much easier to obtain while also convey helpful modeling information [18]. Several semi-supervised learning techniques have been developed in the world of wireless network applications, including Wi-Fi based localization [19], [20], smart city services [21], tracking mobile users [22], etc. These works develop efficient and relatively more robust learning solutions, which both extract high semantic features and require less labeling effort by exploiting both labeled and unlabeled data. While greatly ease the data collection process, the supervision scenario these methods consider is still limited from covering the potential situations of data supervision in practical use.

We consider a border scenario with respect to data supervision, known as weak supervision [18], [23]. Such scenario includes incomplete, inexact, and inaccurate labeling for data samples, which often occur at the same time in data acquisition. We propose a deep generative network based on the generalized expectation-maximization algorithm for UWB ranging error mitigation, which enables weakly supervised learning with coarsely labeled training data. Such method integrates the generalized expectation-maximization (GEM) algorithm into deep learning techniques and obtains error estimation with the help of Bayesian inference. Specifically, the weak supervision is considered to include three categories: 1) incomplete supervision where only a subset of training data is given with labels; 2) inexact supervision where the training data are given with coarse-gained noised labels; 3) inaccurate supervision where the given labels are not well paired to the given samples. All of them suit well the conception of prior
knowledge for a Bayesian model. Therefore, the observed waveform together with the unobserved environment label are viewed as the complete data in the statistic model. The ranging error, accordingly, is modeled as the unknown parameter obtained by the maximum likelihood estimate (MLE) over the complete data. The weakly supervised labels (i.e., incomplete or coarse), in turns, are modeled to provide prior information. Two sub neural modules, referred to as E-Net and M-Net, are adopted to conduct the GEM algorithm in an end-to-end manner. During training, E-Net estimate the environment label while M-Net utilize raw received signal as well as the estimation from E-Net to accomplish the ranging error estimation.

Our contributions are summarized as follows:

- We present a Bayesian model for UWB ranging error mitigation. The model inherits a transparent interpretation and is flexible to scale to different environments.
- We propose a deep generative network based on the GEM algorithm, which combines benefits from both sides and conducts efficient ranging error mitigation on UWB data.
- Our framework is the first investigation that extends DL-based error mitigation methods to the weak supervision paradigm, which greatly reduce the effort in data collection.
- The proposed method based on weakly supervised data contains competitive results against conventional methods with fully supervised data.

The remaining sections are organized as follows. Section II introduces the problem statement and the proposed GEM framework. Section III introduces the implementation of the proposed method in deep learning, consisting of neural modules to conduct the two steps and the parametric objective function for efficient learning. Experimental results on two different datasets are illustrated in Section IV, with comparison to several conventional methods. Finally, a conclusion and future focus can be found in Section V.

II. MODEL FORMULATION

A. Problem Statement

In a harsh environment with obstacles and reflecting surfaces, the received signal at the agent can be written as follows,

\[ r(t) = \sum_i \alpha_i s(t - \tau_i) + z(t), \quad t \in [0, T] \]  

where \( s(t) \) is a known wideband waveform, \( \alpha_i \) and \( \tau_i \) are the amplitude and delay, respectively, of the \( i \)th path, \( z(t) \) is the observation noise, and \([0, T]\) is the observation interval. We will denote \( r(t) \) as \( r \) for convenience in the rest of the paper. The relationship between the true distance \( d \) and the delays of the propagation paths is:

\[ \tau_i = 1/c(d + b_i) \]  

where \( c \) is the propagation speed of the signal, and \( b_i \geq 0 \) is a range bias. Mostly, the range bias \( b_i = 0 \) for LOS propagation, whereas \( b_i > 0 \) for NLOS propagation. Suppose \( d_M \) is the measured distance by the UWB device, the target of mitigation is to estimate the range bias in the specific path and remove from the measurement \( d_M \).

Suppose the ranging error is denoted by \( \Delta d \), where \( \Delta d = d_M - d \). In the following we will show a GEM framework for efficient learning of the estimation of ranging error \( \Delta d \) given the received signal \( r \).

B. GEM Framework

We take the actual ranging error \( \Delta d \) as the unknown parameter to be estimated. From an aspect of MLE, the target is to estimate \( \Delta d \) that maximizes \( \log p(\Delta d | r) \). However, such distribution is hard to obtain due to the complicated propagation environment. Instead, we introduce the environment label \( k \) for the latent variable, and estimate \( p(k | r) \) together with \( p(\Delta d | k, r) \) alternatively. The procedures are conducted by the generalized expectation-maximization (GEM) algorithm. Such environment label can be the LOS or NLOS conditions, different geometric rooms, or different blocking materials for the received signal. The MLE of \( \Delta d \) is then conducted on complete data \((r, k)\), i.e., \( \log p(\Delta d | r, k) \).

With the complete data being \((r, k)\) and unknown parameter being \( \Delta d \), the estimation of ranging error can be obtained from the MLE of the parameter by maximizing the marginal likelihood of the observed data \( r \). Such likelihood can be written as:

\[
\log p(r | \Delta d) \geq \mathbb{E}_{q(k)} \left[ \log \frac{p(\Delta d | k, r)}{p(\Delta d)} \right] - D_{KL} \left( q(k) \mid \mid p(k | r) \right) \\
+ \log p(r) \\
:= \mathcal{F}(q, \Delta d; r)
\]

where \( D_{KL} \) is the Kullback-Leibler divergence. The inequality in the second line is obtained from the Jensen’s inequality, achieving equality iff \( q(k) = p(k | r) \). The inequality in equation (12) can be obtained as follows,

\[
\log p(r | \Delta d) = \log \sum_k q(k) \frac{p(r, k | \Delta d)}{q(k)} \\
\geq \sum_k q(k) \log \frac{p(\Delta d | k, r) p(k | r) p(r)}{q(k) p(\Delta d)} \\
= \mathbb{E}_{q(k)} \left[ \log \frac{p(\Delta d | k, r)}{p(\Delta d)} \right] - D_{KL} \left( q(k) \mid \mid p(k | r) \right) \\
+ \log p(r) \\
:= \mathcal{F}(q, \Delta d; r)
\]

The GEM algorithm seeks to find the estimation by iteratively applying the following two steps:

- Expectation step (E-step)
  \[ q^{(n)} = \arg \max_q \mathcal{F}(q, \Delta d^{(n)}; r) \]  
- Maximization step (M-step)
  \[ \Delta d^{(n+1)} = \arg \max_{\Delta d} \mathcal{F}(q^{(n)}, \Delta d; r) \]
To fulfill the formulation of the objective function in Eq.(12), \( p(Δd) \) and \( p(k|r) \) can be given by prior knowledge, while expressions for \( p(Δd | k,r) \) and \( q(k) \) are required. Since these distributions are hard to be approximated by model knowledge, we adopt techniques from deep learning to accumulate knowledge from data. Specifically, we utilize neural networks as well as datasets labeled with actual ranging error as environment labels to learn their analytical forms.

### III. Deep Learning Implementation

In conventional cases of GEM, the two steps are conducted alternatively by optimizing the analytical solution of \( \mathcal{F}(q, Δd; r) \). However, it is hardly possible in our case as the distribution of waveform data is complicated due to the high dimensionality. Instead of making assumptions on the distributions to reduce the complexity, we construct a neural network to learn the optimization for the two steps. The network structure is illustrated in Fig.2.

#### A. Weakly Labeled Dataset

Suppose we are given a weakly labeled dataset \( \mathcal{D} = \{r^{(i)}, k^{(i)}, Δ\bar{d}^{(i)}\}_{i=1}^{N} \) with \( N \) i.i.d. sample pairs, where \( k^{(i)} \) denotes the label for \( i \)'th environment index, and \( Δ\bar{d}^{(i)} \) denotes the label for \( i \)'th ranging error. Both labels are of weak supervision. In particular, these labels are coarsely gained and not always ground-truth. The specific values of these labels could be ground-truth with noise, randomly mismatched values for other samples, and at default. We take these weak labels to construct prior knowledge for the GEM framework.

Let \( \bar{k}^{(i)} \) and \( Δ\bar{d}^{(i)} \) denote the according estimated environment label and ranging error.

#### B. Neural Modules

In E-step, the optimization of \( q \) is conducted via learning \( \bar{k} \) by the E-Net with parameter \( φ \), i.e.,

\[
\bar{k}^{(i)} = f_φ(r^{(i)}) \sim q(k) \tag{7}
\]

where \( f_φ(·) : r \rightarrow k \) denotes a vector-valued function parameterized by \( φ \), mapping from the observed data \( r \) to the latent data \( k \).

In M-step, the estimation of the ranging error \( Δ\bar{d} \) is obtained by the M-Net with parameter \( \theta \), i.e.,

\[
Δ\bar{d}^{(i)} = g_θ(r^{(i)}, \bar{k}^{(i)}) \tag{8}
\]

where \( g_θ(·) : r \rightarrow Δd \) denotes a vector-valued function parameterized by \( θ \), mapping from the observed data \( r \) to the unknown parameter \( Δd \).

The back-propagation (BP) algorithm for deep learning can avoid the dead-lock problem of the simultaneous optimization over the latent data distribution and unknown parameter, and hence merge the two-step GEM into a whole end-to-end learning network without further alternation. Therefore, the objective function of the end-to-end network with three neural modules can be expressed as:

\[
φ, θ = \arg \max_{φ, θ} \bar{\mathcal{F}}(φ, θ; r). \tag{9}
\]

### C. Parametric Objective Function

The optimization for network learning requires the objective function to be differentiable w.r.t. parameters \( φ \) and \( θ \). Therefore, we derive the analytical version of the GEM objective in Eq.12. Since the last term \( \log p(r) \) remains constant, we derive the analytical forms for the first two terms.

The optimization of the first expectation term \( \mathbb{E}_{q(k)} [\log \frac{p(Δd | k, r)}{p(Δd)}] \) can be conducted via a MLE loss between ranging errors given

\[
\mathcal{L}_{exp}(φ, θ; r^{(i)}, Δ\bar{d}^{(i)}) = \left\| Δ\bar{d}^{(i)} - Δ\bar{g}^{(i)} \right\|^2 = \left\| Δ\bar{d}^{(i)} - Δg_θ(k^{(i)}, r^{(i)}) \right\|^2 \tag{10}
\]

The optimization of the second KL term

\[
−D_{KL}(q(k)||p(k|r)) \text{ can be conducted by the cross-entropy loss between label distributions given as}
\]

\[
\mathcal{L}_{kl}(φ; r^{(i)}, k^{(i)}) = −\sum_{j=1}^{K} p(k^{(i)}_j) \log q_φ(k_j|r^{(i)}) \tag{11}
\]

where the variational distribution \( q_φ(k^{(i)}_j) \) is learned by the network with parameter \( φ \), which can be simply done by empirically calculating the frequency of the output of the. The prior \( p(k^{(i)}|r) \) is estimated empirically from the weak labels \( k^{(i)} \) from the dataset.

Therefore, we achieve the analytical version of the GEM objective in Eq.12 by combining Eqs.(11)-(10), which is differentiable w.r.t. parameters \( φ \) and \( θ \) for the back-propagation (BP) algorithm for network learning.

\[
\bar{\mathcal{F}}(φ, θ; r^{(i)}, k^{(i)}, Δ\bar{d}^{(i)}) = \mathcal{L}_{exp}(φ, θ; r^{(i)}, Δ\bar{d}^{(i)}) + \mathcal{L}_{kl}(φ; r^{(i)}, k^{(i)}) \tag{12}
\]

Since the BP algorithm can avoid the dead-lock problem of the simultaneous optimization over the latent data distribution and unknown parameter, the two-step GEM can be unfolded to an end-to-end learning network without further alternation. The optimization on dataset \( \{r^{(i)}, k^{(i)}, Δ\bar{d}^{(i)}\}_{i=1}^{N} \) can be conducted with the network structure in Fig.2, expressed as:

\[
φ, θ = \arg \max_{φ, θ} \sum_{i=1}^{N} \bar{\mathcal{F}}(φ, θ; r^{(i)}, k^{(i)}, Δ\bar{d}^{(i)}). \tag{13}
\]

### IV. Experiments

Suppose the values of the environment label \( k \in \{0, 1\} \) in our case, where 0 refers to the LOS condition and 1 refers to the NLOS condition. The proposed method, referred to as GEM for convenience, is discussed with different weak supervision scenarios and compared to baseline method to illustrate the effectiveness and superiority.
We compare the performance of our models with other methods on a public UWB database [13], consisting of the received waveforms, LOS or NLOS condition labels, and the actual ranging errors recorded in different indoor environments. The dataset is created using SNPN-UWB board with DecaWave DWM1000 UWB pulse radio module and generated in two different office environments. In the first environment, two adjacent office rooms with connecting hallway is considered, where 4800 measurements in the first room and 5100 measurements in the second. The second environment was a different office environment where multiple rooms, including 25100 measurements in total. The waveform is represented as the absolute value of CIR, with the length of 152. We assign 80% of the data samples for training and the rest 20% for testing, without overlapping between the two sets.

### B. Data Processing and Baseline

We test the algorithm under both weak and full supervision. For the full supervision case, the fully labeled dataset from the database is used, i.e., $D_{\text{full}} = \{r^{(i)}, \hat{k}^{(i)}, \Delta \hat{d}^{(i)}\}_{i=1}^{N}$, with $N$ i.i.d. sample pairs, where $\hat{k}^{(i)}$ denotes the actual label for $i$th environment index, and $\Delta \hat{d}^{(i)}$ denotes the actual label for $i$th ranging error.

For the weak supervision case, we synthesize a weakly labeled dataset $D_{\text{weak}}$ from $D_{\text{full}}$. Specifically, suppose the dataset consists of $N$ labeled samples $\hat{M}_k$ samples with weak environment labels, and $M_e$ samples with weak error labels. The weak label here refers to incomplete, inexact, or inaccurate cases. We define the supervision rate of environment label $k$ and error label $\Delta d$ as

$$\eta_k = \frac{N}{\hat{M}_k + N},$$

$$\eta_e = \frac{N}{M_e + N}. \quad (14)$$

We randomly pollute the data labels with $\eta_k$ and $\eta_e$ by deleting, adding noises, and substituting values with other labels. The proposed method is evaluated under different supervision rates, i.e., $\eta_k, \eta_e \in \{0.2, 0.4, 0.6, 0.8, 1.0\}$.

The classic Support Vector Machine (SVM) method is utilized as baseline method for ranging error mitigation, trained on the full supervised dataset with physical features extracted from the waveform, as suggested in [11], [24]. It can be seen that the proposed method conducts efficient error mitigation under different supervision rates, and still outperforms SVM even with weak supervision.

### C. Results under Different $\eta_k$

While supervision rate $\eta_e = 0.8$ is frozen, the proposed method implemented with different supervision rate $\eta_k$ are compared. Quantitative results are shown in Table I, in terms of root mean square error (RMSE), the mean absolute error

![Network Structure](image)

**Fig. 2.** Network structure of the proposed method, consisting of an E-Net to approximate the distribution of unobserved environment label and a M-Net to estimate the unknown ranging error. The network parameters, $\phi$ and $\theta$ respectively, are guided by the GEM algorithm on a weakly labeled dataset.

### TABLE I

**Quantitative Results of Methods under Different Supervision Rate $\eta_k$ as Freezing $\eta_e = 0.8$.**

<table>
<thead>
<tr>
<th>Methods</th>
<th>RMSE (M)</th>
<th>MAE (M)</th>
<th>Time (MS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unmitigated</td>
<td>0.428</td>
<td>0.291</td>
<td>-</td>
</tr>
<tr>
<td>SVM [24]</td>
<td>0.286</td>
<td>0.175</td>
<td>4.915</td>
</tr>
<tr>
<td>GEM ($\eta_k = 0.4$)</td>
<td>0.135</td>
<td>0.074</td>
<td>1.643</td>
</tr>
<tr>
<td>GEM ($\eta_k = 0.6$)</td>
<td>0.132</td>
<td>0.073</td>
<td>2.368</td>
</tr>
<tr>
<td>GEM ($\eta_k = 0.8$)</td>
<td>0.134</td>
<td>0.072</td>
<td>2.621</td>
</tr>
<tr>
<td>GEM ($\eta_k = 1.0$)</td>
<td>0.123</td>
<td>0.072</td>
<td>0.983</td>
</tr>
</tbody>
</table>

### TABLE II

**Quantitative Results of Methods under Different Supervision Rate $\eta_k$ as Freezing $\eta_e = 0.8$.**

<table>
<thead>
<tr>
<th>Methods</th>
<th>RMSE (M)</th>
<th>MAE (M)</th>
<th>Time (MS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unmitigated</td>
<td>0.428</td>
<td>0.291</td>
<td>-</td>
</tr>
<tr>
<td>SVM [?]</td>
<td>0.286</td>
<td>0.175</td>
<td>4.915</td>
</tr>
<tr>
<td>GEM ($\eta_k = 0.4$)</td>
<td>0.288</td>
<td>0.167</td>
<td>2.122</td>
</tr>
<tr>
<td>GEM ($\eta_k = 0.6$)</td>
<td>0.220</td>
<td>0.122</td>
<td>1.999</td>
</tr>
<tr>
<td>GEM ($\eta_k = 0.8$)</td>
<td>0.134</td>
<td>0.072</td>
<td>2.621</td>
</tr>
<tr>
<td>GEM ($\eta_k = 1.0$)</td>
<td>0.109</td>
<td>0.056</td>
<td>2.045</td>
</tr>
</tbody>
</table>
Fig. 3. The CDFs of the residual errors (remaining errors in range measurements after mitigation) after different mitigation methods under supervision rate (a) $\eta_k = 0.4, \eta_e = 0.8$, and (b) $\eta_k = 0.8, \eta_e = 0.8$. It can be seen that the proposed method outperforms SVM in both cases. The performance of the proposed method is rather robust to the supervision of environment label $k$, with a slight improvement with higher supervision rate $\eta_k$.

Fig. 4. The CDFs of the residual errors (remaining errors in range measurements after mitigation) after different mitigation methods under supervision rate (a) $\eta_k = 0.8, \eta_e = 0.4$, and (b) $\eta_k = 0.8, \eta_e = 0.8$. It can be seen that the proposed method outperforms SVM in both cases, while its performance is significantly improved with higher supervision rate $\eta_e$ for ranging error labels.

D. Results under Different $\eta_e$

While supervision rate $\eta_k = 0.8$ is frozen, the proposed method implemented with different supervision rate $\eta_e$ are compared. Quantitative results are shown in Table II, in terms of RMSE (m), MAE (m), and inference time (ms). It can be seen that methods under all supervision rates successfully mitigate the ranging error to some extent. Methods with the higher $\eta_e$ achieves better performance in error mitigation, while the performance rise w.r.t. $\eta_e$ is more obvious compared to $k$. This implies that the proposed method can efficiently generate information from unlabeled data samples for ranging error information. However, the method is more sensitive to the supervision of ranging error $\Delta d$ than $k$. Thus, the proposed method can achieve a satisfactory performance with a more simple dataset weakly labeled in $\Delta d$ with a rate at around 0.6.

It is worth noting that, almost all the results of the proposed method outperform SVM. This indicates the superiority of learning-based features to hand-crafted features for ranging error mitigation. In addition, the proposed method can exploit the weakly labeled dataset efficiently, while SVM requires fully labeled dataset.

E. CDF Plots for Residual Ranging Error

(MAE), and inference time. It can be seen that methods under all supervision rates successfully mitigate the ranging error to some extent. Methods with the higher $\eta_k$ achieves better performance in error mitigation, while the performance rise w.r.t. $\eta_k$ is not tremendous. This implies that the proposed method can efficiently generate information from unlabeled data samples, especially the inherent information in environment label $k$. Thus, the proposed method can achieve a satisfactory performance with a more simple dataset weakly labeled in $k$ with a rate at around 0.4.
We additionally compare the ranging error mitigation performance in terms of the cumulative distribution function (CDF) for residual ranging errors (i.e., the remaining errors in range measurements after mitigation) under different supervision rates, illustrated in Fig.3-4.

The CDFs of the proposed methods with baseline method under different $\eta_k$ for environment label are shown in Fig.3. It can be seen that SVM trained in a fully supervised manner conducts effective error mitigation, while the proposed method outperforms it by a significant margin under both weak supervision rates for $\eta_k$. While raising $\eta_k$ from 0.4 to 0.8, the performance of the proposed method has a relatively slight improvement.

The CDFs of such methods under different $\eta_k$ for ranging error labels are shown in Fig.4. It can be seen that the proposed method also outperforms the fully-supervised SVM a significant margin under both the considered rates for $\eta_k$. Moreover, the performance of the proposed method has an observable improvement by raising $\eta_k$ from 0.4 to 0.8.

By comparison between the two figures, the proposed approach achieves good results in both cases, while appears to be more sensitive to the supervision on $\eta_k$ than $\eta_k$. This phenomenon is consistent with the intuition that environment label $k$ takes place as a latent variable to give extra modeling information, while ranging error label $\Delta d$ serves as the ultimate estimation target.

V. CONCLUSION

We proposed a weakly supervised learning approach based on GEM algorithm for UWB ranging error mitigation. The approach embedded the signal propagation model in a Bayesian framework, and enabled both efficient and robust estimation of the ranging error. The combination of statistic modeling and deep learning is benefited from both flexibility and efficiency, and enjoys the novelty of conducting such problem in the weak supervision scheme. Although the method is proposed for UWB techniques, it provides a promising methodology for embedding Bayesian modeling in DL techniques, potential to benefit a wide range of learning problems involving a complicated process with latent variables. Future work would be focused on a more flexible framework on radio signal processing, integrating multiple related tasks in a unified Bayesian model.

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