A stabilized discontinuous Galerkin-type method for solving efficiently Helmholtz problems

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Abstract

We propose a stabilized discontinuous Galerkin-type method (SDGM) for solving efficiently Helmholtz problems. This mixed-hybrid formulation is a two-step procedure. Step 1 consists in solving well-posed problems at the element partition level of the computational domain, whereas Step 2 requires the solution of a global system whose unknowns are the Lagrange multipliers. The main features of SDGM include: (a) the resulting local problems are associated with *small positive definite* Hermitian matrices that can be solved in parallel, and (b) the matrix corresponding to the global linear system arising in Step 2 is Hermitian and *positive semi-definite*. Illustrative numerical results for two-dimensional waveguide problems highlight the potential of SDGM for solving efficiently Helmholtz problems in mid- and high-frequency regime.

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