Discrete nonnegativity for nonlinear cooperative parabolic PDE systems with non-monotone coupling

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Abstract: Discrete maximum principles are established for finite element approximations of nonlinear parabolic PDE systems with mixed boundary and interface conditions. The results are based on an algebraic discrete maximum principle for suitable ODE systems.

Keywords: Nonlinear parabolic system, discrete maximum principle, finite element method, nonobtuse simplicial meshes

Mathematics Subject Classification: 65M60, 65M50, 35B50

1 Introduction

The numerical solution of parabolic partial differential equations or systems of equations is a widespread task in numerical analysis, see, e.g., [19, 20, 21]. The discrete solution is naturally required to reproduce the basic qualitative properties of the exact solution, e.g. the maximum principle.

In our recent paper [11], we proved the discrete analogue (discrete maximum principle, or DMP, in short) of the maximum principle for the case of finite element space discretizations for some nonlinear parabolic PDE systems. Besides standard general smoothness and growth conditions, we assumed cooperativity and diagonal dominance for the nonlinear coupling of the equations. Whereas cooperativity seems to be an inherent property behind the DMP, the diagonal dominance (which implies monotonicity of the coupling vector function) is a strong assumption which was only technical. In the present paper diagonal dominance is not assumed, we only require instead that the lower bound of the sums of Jacobians does not deteriorate as t or $|\xi|$ tends to infinity.