Fast simulation of through-casing resistivity measurements using semi-analytical asymptotic models. Part 1: accuracy study

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**Main goal:** To obtain a better characterization of the Earth’s subsurface

**How:** Recording borehole resistivity measurements

**Procedure:**
- Well
- Logging Instrument
- Transmitters
- Receivers
**PROBLEMS**

- **Practical Difficulties:**
  - It is not easy to drill a borehole
  - It may collapse

- **Practical Solutions:**
  - Use a metallic casing
  - Surround with a cement layer

- **Problem solved, but...**
**Problems**

- **Practical Difficulties:**
  - It is not easy to drill a borehole
  - It may collapse

- **Practical Solutions:**
  - Use a metallic casing
  - Surround with a cement layer

- **Problem solved, but...** Numerical problems due to the high conductivity and thinness of the casing
Conductivity and casing width:

\[
\begin{cases}
\delta = 1.27 \times 10^{-2} \text{ m} \\
\sigma_c = 4.34 \times 10^{6} \text{ } \Omega^{-1} \text{ m}^{-1}
\end{cases}
\]

\[\Rightarrow \sigma_c \approx \delta^{-3}\]

First approach:

\[\sigma_c = \alpha \quad \alpha \in \mathbb{R}\]

Case to be studied:

\[\sigma_c = \alpha \delta^{-3} \quad \alpha \in \mathbb{R}\]
**AIM OF THIS STUDY**

- **Develop**: Asymptotic method for avoiding the conflictive part of the domain (casing)

- **Scenarios**: As we are considering axisymmetric scenarios, we can work with two dimensional scenarios

### Preliminary Scenario

```
C A S I N G

ROCK
```

### Target Scenario

```
B O R E H O L E

C A S I N G

C E M E N T

ROCK 1

ROCK 2
```
**Definition**: Let \( u \) be the reference solution. We say an asymptotic model is of **Order** \( n+1 \), if its solution \( u^{[n]} \) satisfies

\[
\|u - u^{[n]}\|_{L^2} \leq C \delta^{n+1}
\]
EQUATIONS FOR THE ELECTRIC POTENTIAL

$$\text{div} [(\sigma - i\delta \omega) \nabla u] = -\text{div} j$$

PRELIMINARY SCENARIO ($\omega = 0$)

Where the solution is expressed as

$$u = \begin{cases} 
  u_e & \text{in } \Omega_e \\
  u_c & \text{in } \Omega_c 
\end{cases}$$

and $\sigma_e$, $\sigma_c$, $f$ are known data.
Asymptotic expansion of the solution:

- In the casing: \( u_c(x, y) = \sum_{n \in \mathbb{N}} \delta^n U^n_c \left( x, \frac{y}{\delta} \right) \)

- Outside the casing: \( u_e(x, y) = \sum_{n \in \mathbb{N}} \delta^n u^n_e(x, y) \)

Where \( \delta \) is the width of the casing and we use the scaling

\[ Y = \frac{y}{\delta} \in (0, 1) \quad \text{when} \quad y \in (0, \delta) \]
One identifies simpler problems satisfied by truncated expansions outside the casing (up to residual terms)

- **Order 1:**
  \[
  \begin{align*}
  \sigma_e \Delta u &= f \quad \text{in} \quad \Omega_e \\
  u &= 0 \quad \text{on} \quad \Gamma 
  \end{align*}
  \]

- **Order 3:**
  \[
  \begin{align*}
  \sigma_e \Delta u &= f \quad \text{in} \quad \Omega_e \\
  u + \delta \frac{\sigma_e}{\sigma_c} \partial_n u &= 0 \quad \text{on} \quad \Gamma 
  \end{align*}
  \]

**Remark:** Second model has already order 3 of convergence due to the flat configuration of the layer
**FINITE ELEMENT METHOD** (Matlab Code)

- Straight triangular elements
- Lagrange shape functions of any degree

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**Domain**

\[ \sigma_e = 0.5 \]

\[ \sigma_e = 2 \]

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**Mesh**

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**Reference Solution**
**Reference Solution**
(in $\Omega_e$)

**Order 1 Model**

**Order 3 Model**

**Definition:** We define the relative error between the reference solution $u$ and the asymptotic solution $u[n]$, as

$$\frac{||u - u[n]||_{L^2}}{||u||_{L^2}}$$
ERROR ANALYSIS

ERROR DEPENDING ON EPSILON IN LOGARITHMIC COORDINATES

Casing Thickness

<table>
<thead>
<tr>
<th>Casing Thickness</th>
<th>0.001</th>
<th>0.002</th>
<th>0.004</th>
<th>0.007</th>
<th>0.013</th>
<th>0.024</th>
<th>0.043</th>
<th>0.078</th>
</tr>
</thead>
<tbody>
<tr>
<td>Order 1 Slopes</td>
<td>0.975</td>
<td>0.956</td>
<td>0.925</td>
<td>0.873</td>
<td>0.792</td>
<td>0.677</td>
<td>0.534</td>
<td>0.382</td>
</tr>
<tr>
<td>Order 3 Slopes</td>
<td>2.990</td>
<td>2.074</td>
<td>1.468</td>
<td>1.440</td>
<td>2.188</td>
<td>2.362</td>
<td>2.148</td>
<td>1.872</td>
</tr>
</tbody>
</table>
Relative error between a solution of degree 10 and solutions of lower degrees

**REFERENCE MODEL**

**ORDER 3 MODEL**

**CONCLUSION:** Error analysis is not relevant once we reach a relative error of $10^{-2}$
NUMERICAL FEM SOLUTIONS

**Domain**

<table>
<thead>
<tr>
<th>$\sigma_b$</th>
<th>$\sigma_c$</th>
<th>$\sigma_{cem}$</th>
<th>$\sigma_{c1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.5</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

**TARGET SCENARIO**

Number of elements = 112
Degree of polynomials = 3

**Reference Solution**
outside $\Omega_c$

**Order 1 Model**

**Order 3 Model**
ERROR DEPENDING ON EPSILON IN LOGARITHMIC COORDINATES

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<th>0.024</th>
<th>0.043</th>
<th>0.078</th>
</tr>
</thead>
<tbody>
<tr>
<td>Order 1 Slopes</td>
<td>0.967</td>
<td>0.944</td>
<td>0.904</td>
<td>0.843</td>
<td>0.751</td>
<td>0.631</td>
<td>0.493</td>
<td>0.365</td>
</tr>
<tr>
<td>Order 3 Slopes</td>
<td>4.764</td>
<td>1.637</td>
<td>1.187</td>
<td>1.291</td>
<td>1.116</td>
<td>1.742</td>
<td>1.911</td>
<td>1.846</td>
</tr>
</tbody>
</table>


Asymptotic models with $\sigma_c = \alpha \delta^{-3}$ $\alpha \in \mathbb{R}$

Physically more realistic models

3D electromagnetic models
Perspectives

- Asymptotic models with $\sigma_c = \alpha \delta^{-3}$ $\alpha \in \mathbb{R}$
- Physically more realistic models
- 3D electromagnetic models

THANK YOU FOR YOUR ATTENTION